Publisert i Linnanmäki, K. & Gustafsson, L. (Eds.) 2009. *Different Learners – Different Math?* Åbo Akademi University Publication No 17/2009

Different Learners – General *Learning Processes in Learning Math?*

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Abstract: The Vasa conference theme poses a question: Different Learners - Different Math? This raises a counter question *Different Learners* – General *Learning Processes in Learning Math?* – which I intend to deal with in this article. Through decades and centuries there has been conducted research on learning and teaching. I mainly focus but a few; the well known Russian researchers Lev Vygotsky and Alexander Luria, and the not so famous Norwegian Mage Nyborg.
I address Luria's theory of the functional units of the brain; Vygotsky's concepts zones of development and word functions in language; and Nyborg's concept teaching model, CTM.
Before I give a brief introduction to these theoretical considerations; I present a case. Tina had significant learning problems, but still she learnt multiplication to a remarkable degree of understanding. I try to describe her learning in light shed by these researchers. Finally I present some examples from an approach to math learning based on these theories.

Throughout decades and centuries, researchers in social sciences have conducted investigations in human development and learning, trying to find general aspects of learning processes. Did they all fail? On the other hand, do we have to recognize Vygotsky's Zones of development, Luria's categories of brain functions, Piaget's assimilation and accommodation and Gardner's multiple intelligences as expressions of something general in mental functions and learning?
In my work, I presume such a generality, and consider theories of learning valid in telling us something about learning in general. Such validity will depend on whether it is possible to apply the theory in designing a learning program for a particular child who is going to learn a particular task; as well as a more general approach presented in textbooks or schoolbooks of any kind. My experiences confirm this, and in this context it will be proper to choose a case where the objective is mathematic learning. What I should do first then, is to demonstrate how Tina did learn multiplication to a remarkable degree of understanding, in spite of her general learning problems. Then I refer to relevant theory, which might provide possible explanations of her achievements; and finally I give some examples of how the same theories also provide approaches possible to apply in designing teaching materials including school books.

Case presentation: Tina, the particular learner.

One day when she was a sixth grader; Tina, a girl with learning disabilities, asked her teacher: "When am I going to learn to multiply?" She had noticed what her peers was occupied with; and was not aware of that maybe such a task would be too difficult for her. The teacher was, but still gave the only possible answer to such a question: "Now, if you want to". Tina already had exceeded what many professionals would hold for her limits, with reference to the IQ test result 52 when she was seven.
With the task such defined, conceptualization of multiplication appears to be prerequisites for training and application of multiplication in calculations. Nyborg's concept teaching model (CTM) tells us to give children primary experiences and verbally express them in a systematic way, taking into consideration certain learning processes and the verbalization both of the generic system concepts belong to and the detection of similarities and differences.
So far in Tina's learning; CTM had proved to be a useful approach; so it was a straightforward decision to choose this method also when multiplication was the concept to be learnt. According to CTM, there were created situations that gave opportunities for an association bond between current problems where multiplication was the way to solutions, and the word *multiplication*[1]. Prerequisites for this learning would

[1] In Norway some would probably use the word "ganging"; which could be considered more easily understood. To choose vocabulary when there are alternative category names is one of the teacher's duties, and the preference should be the one which facilitates learning in being most useful for transfer.

be knowledge of numbers[2] in sets, knowledge of increasing numbers by adding; and similarities and differences between numbers in sets. Advantageous for the task would also be to know how to count in sets; of which two, three, five or ten at a time will be the easiest. In this case, Tina had the required knowledge, as results of former learning, so it was just a question of preparing the proper lessons for her work.
During the first lessons the task was to find the total number when objects were organized in a number of sets with equal numbers of objects. A reasonable way to verbally express such a situation would be: "The sets have equal numbers, and then we can multiply"[3]. The calculation can be by means of counting; count the number in each set and the number of sets, and use a calculator or refer to a multiplication table written or in someone's head to find the exact number. I will stress the importance of, in such a sequence of tasks, varying the kind of objects in the sets, the number in each set, and the number of sets to make sure the mental connection established is between the numbers of sets equal in number of objects and the expression "we can multiply, because the numbers in the sets are equal".
The next step is to organize a group of sets equal in number next to a group of sets where at least one of them has another number than the others. The task, then, would be to point out the group in which it is possible to find the number by multiplication. Again, the learning processes depend on use of language. Expressions like "in that group the number is equal in each set, so we can multiply"; "it is true that we can multiply to find the number in that group, because the numbers in the sets are equal", and contrary "it is not true that we can multiply to find the number of that group, because the sets are not equal in number" will establish verbal consciousness of these processes, and hence facilitate learning. There should also be a sequence of tasks like this, to avoid invalid inferences.
To consolidate the learning, another sequence of tasks is required in order to establish verbal consciousness of the similarities between groups or situations where multiplication is a relevant way to solve the problem. Again we present two groups of sets, different in the kind of object, the number in each set and in the number of sets, but similar in that the sets within a group have equal numbers. The question is to tell any similarity of the two groups. And any similarity should be praised by the teacher, but one should make certain that the issue of similarity in number in the sets in each group, and the consecutive possibility to multiply is detected and expressed. Such a verbal expression of similarities tells us that the learner has learnt the concept of multiplication to an extent including transfer to new situations where multiplication would be a relevant way to solve a number problem.

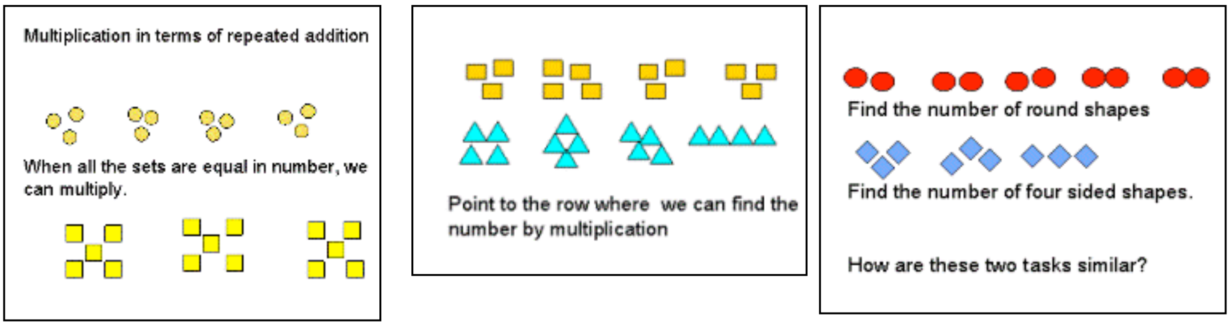

Figure 1.
Examples, demonstrating one task for each of the three processes described in the text (establishing associations between category members and the category name; distinguishing between members and non-members of the category and verbal consciousness of similarities between different category members) [Note, that for children we should use real things, not pictures.

[2] Included in this is the awareness of numbers as a prerequisite for the abstraction of this feature. This is obviously a problem for many of the children who earn some diagnosis telling that she or he has math problems. To apply the word number together with the "number word" and say the number two, the number three and so on is often considered more difficult, but in fact it provides children verbal tools for directing their attention to the number, and hence facilitate the process of abstraction for these children.
[3] Note that in this case only the context repeated addition is in focus. Other kinds of contexts where multiplication would be relevant for solving problems were not dealt with at this stage.

After this sequence of lessons, adapted to the situation through them, Tina started training the multiplication tables in different ways. There were little, but still some, activity of writing in books, and a lot of games on the screen and on the table. Games on the table, together with someone else, provide situations where we need language. This influence the processes of verbal consciousness, and hence the learning as a whole.
By the end of the year, Tina knew how to multiply, she knew some of the multiplication tables, and she mastered to use aids like written tables or calculator when necessary.
The teacher was not confident that she really understood the issue of multiplication, and to evaluate she gave Tina a sheet from the textbook (Fig 2), and provided no help. She was extremely surprised by what followed. Tina took a few seconds to sort out, ant then filled in the numbers at the top of the page, on the lines dedicated for it. She first reached 90 at the second last line, and understood that it was not correct. She revised, and at once she discovered the error she had made and corrected it. Next, she read the consecutive task, remained thinking for a few seconds, and then quickly filled in the upper row of numbers at the top of the page. The table she so designed gave her the answers, and she quickly and without hesitation fulfilled the task. She was proud, and so was the teacher.

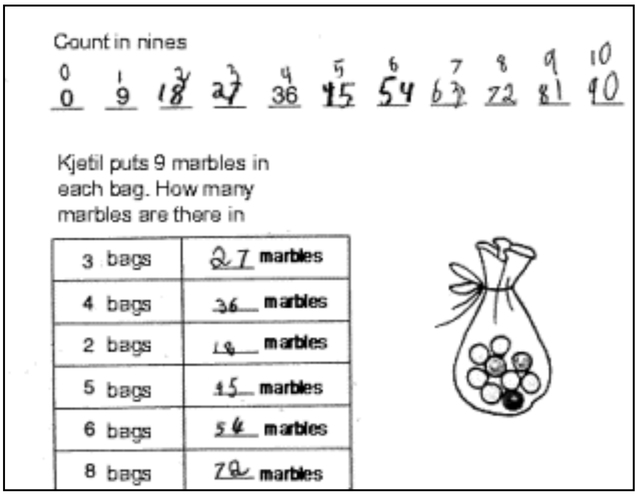

Figure 2. Tasks taken from Abakus 4. The student's problem solving is described in the text.

This is neither the story of a wrong diagnose or of a talented teacher, but a result of systematic application of learning theory, and we shall now have a brief look into a small part of that theory.

Theory, intended to shed light on learning processes.

In the theoretical considerations I will first point to some important implications of Alexander Luria's works. Lev Vygotsky also has influenced research and practice in the field at stake, all around the world, and I will return to a few of his concepts. The former Norwegian professor Magne Nyborg claimed to be eclectic in his work. His research and analyses were heavily loaded with the inheritance from both Luria and Vygotsky. The three of them had different points of view; Vygotsky and Luria were descriptive each in their way, while Nyborg was aimed at practice, and particularly at improvement of practice. Let us have a brief look into a few of their conceptions.

Functional Units of the Brain; Alexander Luria.

Alexander Luria was aware of being in a young branch of science in his neuropsychological work in the sixties and seventies. As he reported his findings, based on brain lesions and post mortem investigations, he also claimed that future research methods, which he and his contemporaries had already vague ideas of, could prove some of them to be wrong. My judgment is that there is still something in it for us, when our objective is to understand mental functioning and learning. Instead of that time's theories about a narrow localization of mental functions in the brain, Luria brought arguments for a theory of dynamic localization

and changes in cerebral organization of higher psychological processes in course of development. Luria found evidence for different functional units of the brain, and identified three principal of them; a unit for regulating tone or waking, a unit for obtaining, processing and storing information from the outside world, and a unit for programming, regulating and verifying mental activity. He pointed to these units' broader localization, primarily considering them to consist of concertedly working brain structure groups (Luria, 1973). He also stated their participation to be necessary for any type of mental activity.
He found each of the units to be hierarchical in structure, and pointed to three levels in respect of three cortical zones. He considered the projection area, receiving and sending impulses, to be the primary unit; the secondary, projection – association, broadly processes incoming information; and the tertiary is an overlapping system responsible for the most complex forms of mental activity, requiring concerted participation of many cortical areas.
In the following, I will focus the part of Luria's work where he describes the three functional units. It is important, then, to state that these units work in close cooperation between the different levels of the cortex.
Relevant questions posed to this, applying to math learning, could be how to present new tasks in ways contributing to alertness and persistent attention– allowing to process information and conduct relevant acts, evaluate and monitor them and be able to solve the problem in an efficient way, like Tina did when she demonstrated her competence in problem solving requiring multiplication.

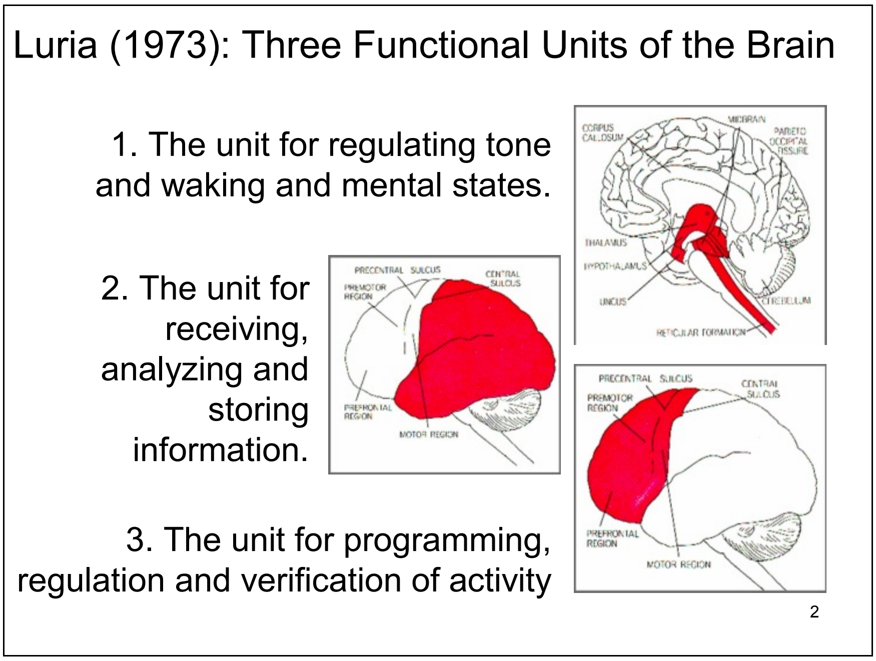

Figure 3. The three functional units of the brain, described by Luria. (Luria 1973)

The first functional unit – sources of activation.

The first requirement for human mental processes; at least those dealing with receiving information; is the waking state.

> *It is only under optimal waking conditions that man can receive and analyze information, that the necessary selective systems of connections can be called to mind.*
>
> *Luria 1973, p 44*

Luria continues to point out how properly directed mental activity is impossible without a certain level of cortical tone. This unit has the structure of a non-specific nerve net playing its role in the regulation of the state of cortical activity and the level of alertness.
The discovery of the reticular formation was a progress in describing these important sources of activation. Luria includes three primary sources: the metabolic processes of the organism; the orienting reflex processes, and a learnt, complex basis for intentions and plans; not far from motivation, I would like to add,

though Luria did not put it that way. This composition of primary and learnt elements is important, and Luria refers to it as having

> *.. a double relationship with the cortex, both influencing its tone and themselves experiencing its regulatory influence.*
>
> *Luria 1973, p 45*

The second functional unit – the unit for receiving, analyzing and storing information.

The primary function of this unit is the reception, coding and storage of information. (Op. cit. p 79.) Probably one could add that we must understand *information* in a broad sense – as everything perceived from sources outside as well as within the person. While the first unit works in accordance with principles of gradual changes, this unit obeys the "all or nothing" rule, by receiving discrete impulses and relaying them to other groups of neurons. The functions belonging to this unit have a high modal specificity, adapted to respective sensory information. This implies primary processes in the projection areas of cortex with the highest load of modal specificity. The surrounding system superposed above them, with a secondary function, has a lower degree of this specificity, as its composition includes associative neurons to combine incoming excitation into functional patterns. The tertiary functions of the second unit integrate excitation arriving from different analyzers. They are multimodal in character, and respond to general features. They also possess a higher degree of lateralization than the secondary zones.
Luria described the second functional unit in terms allowing readers to identify the categories declarative and procedural knowledge[4]. So did Das, Kirby and Naglieri in their development of the PASS theory of learning (Das et al 1994), though their analyses referred to the processes where these categories are activated. So did also Nyborg, and his main focus was how to take advantage of them in analyses of learning processes of the two kinds, which he pointed out to be different in some important respects. I will return to his analyses, but here just mention that the organization of lessons for Tina followed such analyses, considering learning processes concerning respective declarative and procedural knowledge one at a time, work with the declarative knowledge of multiplication before starting work with the procedural. This is, of course, a simplification; as these categories of knowledge cooperate through most learning processes possible to describe.

The third functional unit – the unit for programming, regulation and verification of activity.

The organization of conscious activity is linked with the third of the functional units of the brain, in Luria's terms. In more popular terms we could call it the "director" of mental activity.

> *It is these portions of the brain.., which play a decisive role in the formation of intentions and programs, and in the regulation and verification of the most complex forms of human behavior.*
>
> *Luria 1973 p 84*

There are "two-ways" connections between this unit and both of the two others. While it is "charged" to the appropriate energy tone by the first functional unit, it also contributes "back" by offering strength to certain attention processes, and inhibiting others. I find it worth to underline Luria's statement that the third unit plays an essential role in regulating the state of activity, changing it in accordance with complex intentions and plans formulated *with the aid of speech* (my emphasis). We can see this as an argument also for Nyborg's application of speech into teaching programs, as we shall see.

A sidelong glance at Tina.

Initially, Tina had an extremely short attention span. She could work through 5-10 minutes, and then she would be "tired" with no motivation for further effort. In such a case, we often recommend lessons as short as that. What we saw with Tina, was, that as soon as she got verbal tools for directing attention, she could

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[4] Simultaneous and successive processing research is reported by Das (1972) and Das, Kirby and Jarman (1975, 1979). Current to this is the Norwegian professor Magne Nyborg's application of the same categories in his analyses of learning processes (Nyborg, 1985).

work as long as the activity was within her zone of actual or proximal development (Vygotsky 1962). As soon as the task required knowledge exceeding her limits, she immediately "lost" her attention. The verbal tools to which I refer are the same as already mentioned: words denoting generic concepts like shape, size, position, place; and verbal expressions of similarities and differences. In learning processes concerning concepts, this will count for specific or selective similarities. A possible inference would be that such verbalization, and the verbal consciousness it facilitates, involves the third functional unit in ways that influence attention; *the regulatory influence*, as stated by Luria, quoted earlier. This would be in processes, descending from a higher function to a lower.

In the PASS theory, the planning processes are connected to the third functional unit. Das, Naglieri and Kirby in their exploration of the PASS theory, highlight the importance of the planning processes in learning in general, and particularly their influence on attention. The successful part of Tina's story could be possible to interpret as an example of this.

Lev Vygotsky; language development.

Two concepts from Vygotsky enlighten important spots in the field, and should supply the frames that I intend the analyses in this article to be within. I shall only briefly refer to them. *Zone of proximal development* (Op cit. p 103; often referred to as ZPD) is the most cited, and it has had an important influence in different fields concerning education. The field of Dynamic Assessment has developed from this concept (Haywood & Lidz, 2007; Lidz & Gindis, 2003), as well as the concept of learning potential (Hamers et al, 1995; Haywood,1988; Feuerstein et al, 1986).

> *Resting on the foundations of Vygotsky and Feuerstein, dynamic assessment can be defined as the creation of a zone of proximal development, within which the assessor provides mediation of cognitive processes to promote higher mental functioning of the learner. In curriculum-based DA, this occurs in relation to specific curriculum content.*
>
> Lidz 2007 (II)

Due to my experience I consider Tina's story to be an example of how it is possible to take this a step further; not only to assessment, but to a kind of intervention that contributes to a change in the person – from the proximal zone of development to a new actual zone of development. And we can hear the bell ring in our ears: "*What the child can do in cooperation today, he can do alone tomorrow"* (Vygotsky, op.cit. p. 104).

Though less known than the concept of ZPD, I consider Vygotsky's consideration of language development in terms of *word function from nominative to also include a significative function* not less important.

> *In the beginning, only the nominative function exists; and semantically, only the objective reference; signification independent of naming and meaning independent of reference, appear later, and develop along the paths we have attempted to trace and describe.*
>
> *Vygotsky (1962), p 130*

To provide aids for the action of developing word meanings independent of reference, also could be conceived as means to create zones of proximal development concerning certain kinds of learning, including mathematic learning in some respects. It would be wise to pay attention also to the first part of the phrase; the nominative function of the words. To give words a nominative function, it takes an object to name. And to facilitate the path through the significative function, it takes a series of objects belonging to the same category, and the child labeling them all with their common name. That is how Tina learned multiplication as a concept – by applying the word multiplication in its nominative function, denoting situation by situation relevant to name multiplication. The learning also included discriminations between situations where multiplication could solve the problem, and where it could not, and the verbal expressions of partly similarities between the contexts proper for use of multiplication. That moment the word multiplication became a signifier for Tina, embedded with meaning independent of reference; it allowed her to use multiplication as a tool, even if she would not have been able to explain it.

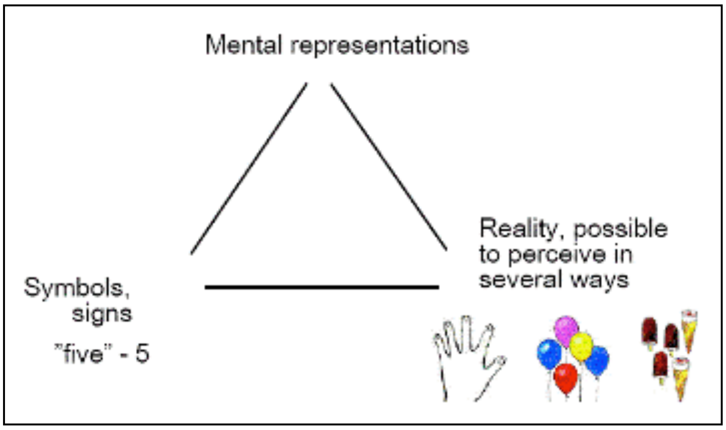


Figure 4. Aristotle's semantic triangle

This distinction between word functions stated by Vygotsky is akin to Aquinas' exploration of how mental representations changes from knowledge of the particular to general knowledge; so this is not novel considerations. But still, there is something in them for us, when we try to understand learning.

What has come to be referred to as Aristotle's semantic triangle (Figure 4), also could demonstrate this (Aristotle, 2004; O'Callaghan, 1997, 2003). Though a conception debated and refuted (Brower-Toland, 2004; O'Callaghan, 2003) it certainly has something to add to our consciousness that it is beyond a person's control what the words in one's utterances awake of understanding in those who receive them. To remain for a moment with ancient writers; it could be relevant to note Aquinas statement that

> *...the singular is known according to sense, while the universal is known according to reason*
>
> Aquinas

In his interpretation of the philosophy of Aquinas, De Wulf (2008) states that humans know of something in two main categories of knowledge; knowledge of particular objects and its forms, and abstract and general knowledge.

The semantic triangle, thus, could visualize this if we put an arrow from the reality, consisting of singular objects and events, to the mental representations consisting of abstract knowledge of the universal or general. When we perceive each singular, and verbalise the name of its category, one after the other, we are more or less compelled to detect their partial similarities. From this abstraction process universal knowledge emerges. In the words of Helen Keller it sounds like:

> *Suddenly I felt a misty consciousness as of something forgotten, a thrill of returning thought, and somehow the mystery of language was revealed to me.*
>
> Helen Keller, 1902

Such general knowledge enables us to reason regardless of the presence of a singular; free of context. In my ears, Vygotsky and Aquinas sound in some important respects alike, and maybe it is not too risky to conclude that they have both understood more than most of us about human consciousness and cognition. This matter, which both of them includes in their writings, Keller describes in her narrative of her own experience.

Generalization processes in Tina's learning.
Let us again, for a moment, turn our attention to Tina; this time with a request for this kind of processes in her development. Her learning problems were so serious that the generalization process initially seemed to be impossible. When she was asked about similarities or even likeness, she did not understand, and was immediate out of the context. But after systematic work she required the needed mental conditions, and was able to detect and express partial similarities. Coincidental she changed her behaviour towards social interactions. While she had been playing alone most of the time so far, she became more approaching towards peers; both in respect of addressing them verbally and by joining games in the breaks. A year after, she one day came to school with a new coat and new shoes. Proud, she showed her new things, and there was a small talk between her and her teacher. She started to work, and suddenly she interrupted, and started the following conversation:

> *Tina: "They are alike!"*
> *Teacher: "Oh! What? Who are alike?"*
> *Tina: "The shoes and the coat".*
> *Teacher (looking carefully at the shoes and the coat, but unable to discover any similarity): "In what respect are they similar?"*
> *Tina: "They are similar in being bought in the shop* Funny Kids*".*

This does not seem to be an important similarity; what is important, is for this girl to have acquired a new strategy to discover and understand the world around her. My judgement is that without this new strategy, even though it was not verbal conscious in most of the situations, she would not have succeeded in learning multiplication like she did. Her learning was depending on a word function signifying meaning, in the way stated by Vygotsky, and the work through that end included creating a zone of proximal development for generalization processes.

Nyborg's analyzes of learning processes; and analytic coding as a basis for them.

The Concept Teaching Model (CTM).
The same distinction between knowledge of the particular versus knowledge of the general is important in the former Norwegian professor Magne Nyborg's writings. It is comprised in his Concept Teaching Model (CTM); in fact the model is designed to facilitate learning processes concerning acquiring general knowledge – which is the core of concept learning (Nyborg, 1985, 1994 I, 1994 II). Nyborg talks of knowledge in terms of images, concepts and concept systems; where images are knowledge of the particular[5], concepts knowledge of the general, and concept systems conceptual structures where super ordinate categories comprise several subordinates. It is easy to think of a system of fruits; consisting of apples, pears, plums, oranges, bananas and so on. When we know about fruits, knowledge of each kind of them will be represented in our long term memory in terms of knowledge of similarities and differences between them; a concept of the category. Apples have some particular similarities which makes us put them into the same category, and at the same time several differences, of which we are not concerned. They can, for instance, be different in color, shape, size and taste, and still we are sure it is an apple, when we see it.

In the Concept Teaching Model (CTM), all these three "levels" are present. When the intention is to teach someone a concept, the primary task is related to images; to prepare conditions for the learner to learn several images of the corresponding several examples from the category he or she is learning about. If the category is groups in which the number is three, we should prepare contexts, one after the other, where the task is to count, find the number of objects in the group, and express it: *It has the number three.* By adding the verbalization into the context, we compel the person to apply the word in a nominative function (Vygotsky), and the learning is of this particular group of whatever objects we choose, connected to the utterance *the number three.* So far, we do not know whether the learner connects the number to the phrase, or if the association goes to the kind of objects, their size, color, or

[5] The word images could mislead us to connect it to visual perception. Still, it denotes mental representations of particular phenomena, regardless of the sense through which it is perceived.

what there could be of possible options in the particular context. Hence, we must create a new context, with a group of another kind of objects, again say: "it has the *number three*" and ask the learner to say it as well. Again the learning is of the particular group connected to the utterance *the number three.*

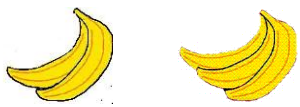
This group of cars has the number three. *The bunch of bananas has the number three .* *The pile of blocks has the number three.*

Figure 5. Examples of tasks to establish connections between numbers of objects and the expression *the number three*; which are tasks from the first part of the CTM.

Initially there were objections to this kind of verbalizations. It was considered proper to emphasize verbal expressions, but within some limits of "natural language". You do not find people who speak like this: "the number three". That kind of objections has faded out, as more and more professionals experience the importance of applying the language in this way in a learning situation. For those who venture, it becomes obvious that we need this kind of expressions to facilitate for the learner to direct his or her attention to the current feature; in this case numbers. In that way the focus on the number is maintained, and the attention to any other feature of the objects suppressed. Still, a number of contexts like this are required, allowing the learner to detect the exclusive similarity between all these groups – in spite of being different in most respects, they are similar in having the number three[6].

Hence, in this first part of concept learning (learning of the general), there is a learning of the particular connected to language, and the verbalization also of the conceptual system it belongs to (or the super ordinate category) facilitates the generalization processes.

The next part deals with learning differences, to make sure the generalization does not go too far. In case of learning a concept of the number three; the distinction will have to be between the number three and any other number, and with a special focus on the numbers two and four, which are the numbers most easily to confuse. Again, the teacher's role is to create contexts where the learner will have to distinguish a group containing three objects from groups with other numbers, and add verbal expressions like ***this** group has the number three*.



Which of the banana bunches has the number three?

Which of the block piles has the number three?

Figure 6. Examples of tasks, concerning learning of differences between objects that belong to the category, and objects that do not; when the objective is to learn a concept of the number three.

Already in the first part of the learning progress described, there is an unconscious detection of similarities. In the third part, this learning is strengthened by verbal consciousness, simply by pointing out and put into words the present similarities. In this case: *They are similar in having the number three*.

[6] How to express this in the most useful way is not an easy decision. When I say about groups that it *has* the number three, rather than *is* the number three, it is to maintain the issue that the group and its objects is the "real thing", the number is a feature of the group. We also could hold the number for "the real thing" and say *this is the number three*, but that would put a demand of more complicated abstractions on the learner.

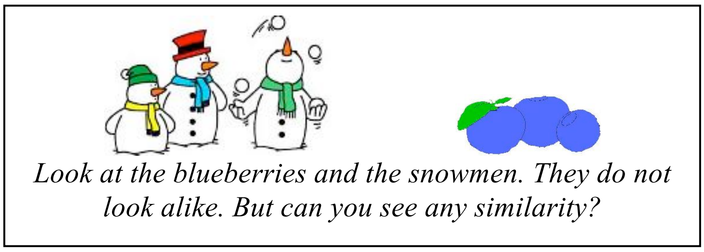


Figure 7. Example of task concerning detection of partial similarities and the expression of verbal consciousness.

When the learner is able to discover and verbalize partial similarities like this, the concept is learned, and the word function has changed from the nominative; naming each object, through the significative; naming a category to which certain objects belong.

Long term memory structures.

In his model of different long term memory structures, Nyborg held cognition or knowledge in the terms described, to be one out of three; the others were *skills* and *dispositions* for emotional and motivational activation (Nyborg 1985, 1994 I, 1994II). This distinction between skills and knowledge, or procedural and declarative knowledge, which are terms more frequently used, is subject to several writings by several authors, like mentioned before. (e.g. Das et al ,1975; Downing, 1979; Hiebert, 1986; McNamara 1994). Figure 8 demonstrates distinct differences between these two kinds of knowledge. While knowledge is organised in a hierarchical way, skills are organised in sequences.

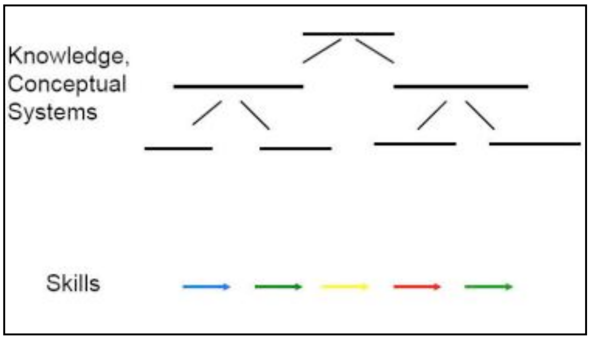


Figure 8. Knowledge and skill structures

According to their distinctive structures, there are also different learning processes concerning knowledge and skills. While concept learning is referred to above, I will give a brief presentation of learning processes in skill learning in the following.

Nyborg identified three phases in skill learning (Nyborg 1985, 1994). These are identical with those described by Downing (1979):

1. Cognition phase – learning about the skill e.g. by observing others perform the skill, learning prerequisites of several kinds.
2. Imitation phase; "try and fail", by self-monitoring or guided by others
3. Exercise towards automation, when the skill can be performed with a minimum of attention to it.

When the learning task is multiplication, as in the case of Tina, the cognition phase will comprise initial learning like identification of numbers and number concepts, to count one by one, to count two at a time, three at a time and so on. To learn a concept of multiplication, as mentioned earlier, also belongs in this phase. In math teaching, this phase has often been neglected, even with the last decades' impact on understanding and language. Meeting children with math problems, this is often uncovered to be a source of learning difficulties. Not until the needed prerequisites are provided should there be any impact on exercises and training. And vice versa; when exercises are rooted in conceptual understanding, children demonstrate motivation to extents that are not easily provided however exciting tasks we can manage to create.

Analytic coding –a set of abstractions.

Nyborg pointed out a category of processes encompassed in most types of learning; analytic coding. Those who use Whecsler's scales will be familiar with the task: tell all you know about....; which is a task demanding processes of analytic coding. The ultimate answer to such a task is in terms of shape, colour, size, position, and so on. The categories needed for such analyses Nyborg regarded basic in certain respects, and the knowledge of them, he called basic concepts, structured in basic conceptual systems. Figure 10 gives a brief overview of the most important of them. You can find this more elaborated at www.pedverket.no, in one of the English sub pages. Figure 9 has an example of a task where these conceptual systems are prerequisites. Point at the entity that has a round shape and a large size compared to the one next to it.

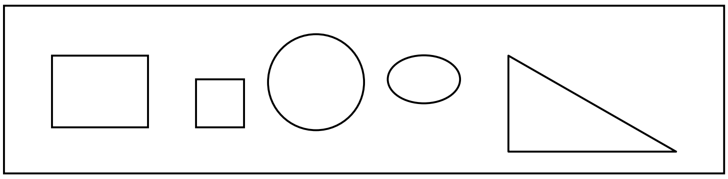


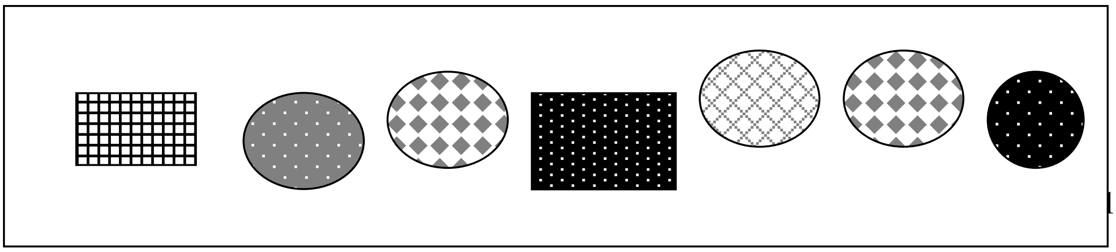
Figure 9. Point at the entity that has a round shape and a large size compared to the one next to it.

Basic Conceptual Systems
Prerequisites for abstraction processes, like analytic coding.

Shapes	Colours
Position	Size/ length/ height
Place, position	Functions
Numbers	Wholes/ parts of wholes
Sounds	Materials
Properties of materials	Patterns
Temperature	Weight
Taste	Smell
Alive/ not alive	Motion/ direction/ speed
Changes	Time
Value	

Figure 10. Nyborg never considered a list like this to be final. He always looked for supplements, other concepts having this function: to allow classification. This overview is simplified compared to the detailed explorations you can find in Nyborg's writings as appendix in his books (Nyborg 1985; Nyborg 1 and II 1994). It could be ordered in other ways, as well.

Figure 11 has another task, more complicated than Figure 9.



Figur 11. Point at the entity that has a round shape, chequered pattern, and is placed to the left

The word abstraction has a number of meanings in literature; here it labels the process of dividing a whole into its parts, also in respect of directing attention to one aspect with, or one property of, the whole, disregarding for the moment the other[7].

A chain of abstractions, then, could be the multiple classification of a cup like this: "*This cup consists of two parts; the bowl and the handle. The top and the bottom have round shapes, as we can pull a finger along the edge, round and round. The handle has a curved shape. The cup is made of stoneware; it has a blue colour, and a dotted pattern. It has a great size compared to an eggcup, and a small size compared to a mug. It has a large weight compared to a ball of wool, and a small weight compared to a plumb. We use it for drinking, most often for drinking something hot. When we use it, it has its place on a table, and to be stored it has its place in a cupboard*". Abstractions of this kind require analyses, or we could say they are analyses.

All three above tasks require even more basic processes, like consciousness of partly similarities and differences, as well as discrimination. In the first task (fig 9), it is sufficient to abstract or direct attention to the shape and the size of the figures. That implies a discovery of similarities and differences between the figures. The task in figure 11 also implies a discrimination process of the figure having a round shape and chequered pattern, but not place on the left of a figure with round shape and dotted pattern, in order to exclude it. In addition, one needs to identify and exclude the figure having a four-sided shape and chequered pattern, in spite of its place on the left of the round shape with dotted pattern. Such analyses can be made simple or complicated, depending on the intention.

We can also create games compelling the children to use these processes, e.g. a Lotto-game (figure 12), where we give the children sheet with figures, and pick up and describe small cards with one figure on each. The instruction can be:
"Who has a round shape with a blue colour?" The lucky one, then, also have to describe: "I have a figure with a round shape and blue colour (unfortunately, the colours will not be of any help in a black-and-white print).

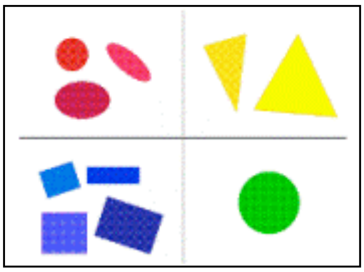


Figure 12. ANNA game for analytic coding processes

[7] This would be the same definition of the word abstraction as qouted by J.S. Peirce, in his work "On a new list of categories": *The terms "precision" and "abstraction", which were formerly applied to every kind of separation, are now limited, not merely to mental separation, but to that which arises from* attention to *one element and* neglect of *the other.* (Peirce, 1868)

Children do like these tasks, and in my practice I regularly see how children with low self-esteem raise their shoulders and experience their own competence, when we have provided tools for managing tasks like this, and they succeed on their own. And even more; for children with more severe learning difficulties, specific or general; when they acquire tools for such analyzes, it looks like you turn on a knob, and something starts going. Quite a number of teachers have made experiences like my own: these processes do change the children's strategies and styles, and enhance their learning potentiality. Still, there is a lack of research on these processes, to judge whether statements like this is valid; to the benefit for children in need for aids to become independent and autonomous persons, able to be directors of their own lives.

Tina and the analyzing processes.

Earlier, I have referred to Tina's problems with generalizations. It might not be a surprise, when I tell that exactly such analytic processes were the key also to the generalization. Only after months with short exercises of analysis with as simple objects as required, Tina mastered the generalization – and was able to use the language not only in its nominative word functions, but also with the words signifying general meaning, independent of context. This is a considerable shift in a person's mental operations, which justifies the amount of work to get there is such a case. Probably, it could have been easier, if methods like CTM were more commonly applied in kindergarten and schools.

There are reasons also to claim, that learning problems concerning math as well as reading are possible to counter, when CTM and activities of analytic coding are considered a part of initial training in education in a broad sense. This is the reality in scattered schools and some small regions in Norway; though not systematically reported so far.

Why? Let us ask Luria.

Luria's writings about the functional units of the brain can throw light on this, and help us understand. Luria talks about ascending and descending processes, both between the three main functions attention, information processing and executive functions, and between levels of functions within them. Imagine a situation where we ask the children to describe, let us say the three round shapes in the ANNA game in figure 12. Before anything can happen at all, the first functional unit has to come to work, to direct their attention to the objects. If a child is able to describe it in terms of shape, colour and number, like this: Here are round shapes having a red colour and the number three (of them); it depends on successive and simultaneous processing, and not less on the "director" in the third functional units, using verbal expressions to compel the attention towards feature after feature possible to discover and express. Once there is established such a descending process from the third unit's"director" to the first unit's attention processes, something happens. Maybe we could use the metaphor of treading a path. Each time such a process is going on; the path becomes more clear and visible; so also with these processes. When they are conducted intentionally, with the aid of speech, a number of times, they continue to work on their own. I have been in short of an explanation to the effect I have regularly seen in children for whom we succeed in providing verbal tools for analytic coding, and guide them into that kind of processes. I think Luria's writings contribute to such an explanation. Research with newer technological aids like fMRI to investigate the neural activity corresponding with these processes, could provide deeper and more valid knowledge about these matters.

It remains to show how these elements of theory can direct teaching aids like schoolbooks and a broader approach to which it belongs; to math learning. The elements I will point to are systematic use of CTM (Concept teaching Model according to Nyborg), and analytic coding. Such an approach will have to rest on initial task analyses concerning math learning. In his writings Nyborg presented a task analysis concerning math learning. This is presented in figure 13.

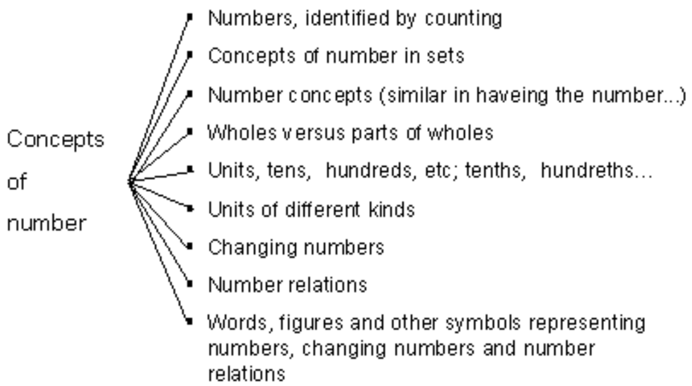


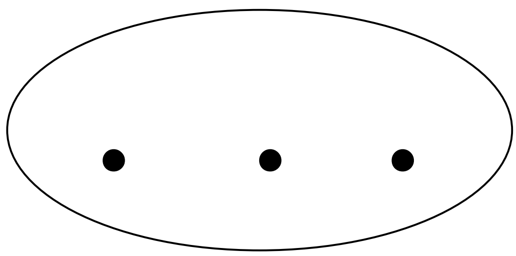
Figure13. Some important conceptual systems concerning mathematics

Teaching materials design according to these theoretical considerations.

Each country has its curriculum, where the actual math knowledge and skills which children are expected to learn at different stages is listed. A teaching approach will have to be according to such curricula. Designing a teaching approach means identifying knowledge and skills encompassed in the curriculum sentences, and to describe ways to provide contexts which facilitate learning of them.

Abstraction of numbers.

A possible inference to draw based on the theoretical considerations above; is that the abstraction of the number, and verbal tools to facilitate it; is of great importance for children’s mathematical understanding and meta-cognition. Number is what we find when we count something. Hence, we should provide contexts for learning, where the children verbalize this in connection with their acts of doing so. The following example can show this, but it is important to remember that the children will need real objects, not just drawings or even pictures of them.



- *Count, and find the number of dots in this ring.*

– *The number is three.*

- *It is! You managed that well.*

The following figures show how this can be reflected in tasks in the math book.

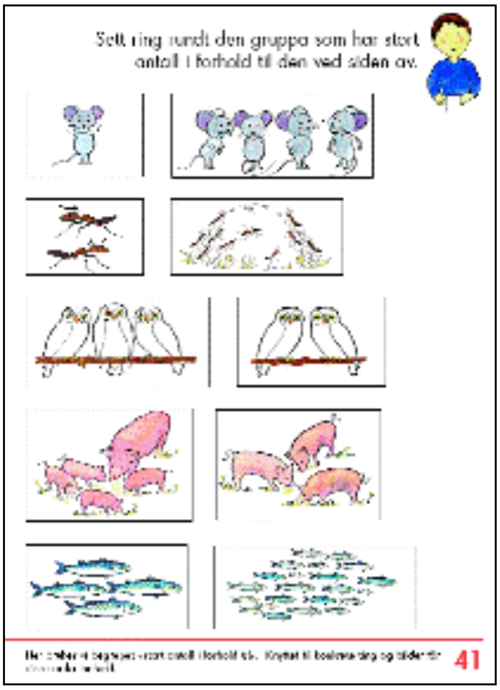


Figure 14. Put a ring round the group having a great number related to the one beside.

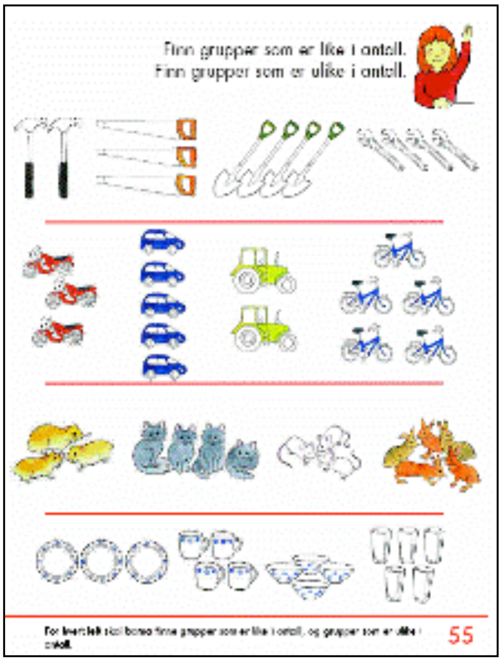


Figure 15. Find groups which are similar in number.

Subtraction.

"Minus" is a difficult entity for a lot of children. This is also a concept, and the CTM provides frames within which it is possible to design a learning progress, taking into consideration to establish a bond between the word and the contexts which are relevant to name so, to distinguish contexts relevant to name subtraction from those irrelevant; and finally to verbalize partial similarities between contexts, for which the word subtraction would be a proper name. The initial subtraction context is connected to the act of decreasing the number by taking away some of the items in a set. The first part of CTM will be to establish a connection between the acts of removing items and expressions like *I remove an item, and then the number decreases.* Children with special needs will need several situations in which they remove objects from a group and express what they do: *I remove..., and then the number decreases.* The situations should include both acts of removing one item, and acts of removing other numbers from the set.[8]

The next part of the CTM regarding conceptual understanding of subtraction would be to present two sets, and remove items from one of them. The child is then asked to point out the group where items were removed, and the number decreased, and to verbalize this. Again children with problems will need several tasks. Finally, there should be several situations allowing the children to detect and express partial similarities between sets similar in the fact that items have been removed from them. Only after such a learning progress, there should be work in a math book. In that way the words becomes signifiers; and get their significative function; hence the children will be independent in application of their knowledge and transfer to new situations.

[8] Prerequisites would be to have learnt concepts of numbers in sets, and for the following to have learnt a concept of symbols as well.

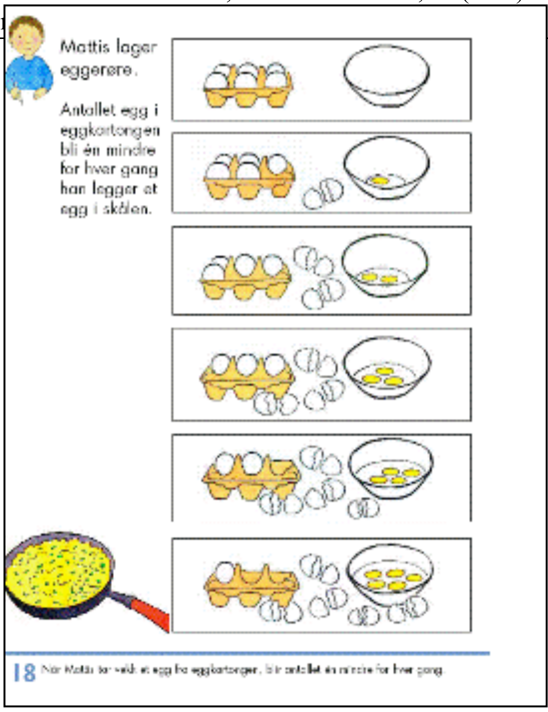


Figure 16. Mattis makes scrambled eggs. The number of eggs in the carton decreases each time he puts an egg in the bowl.

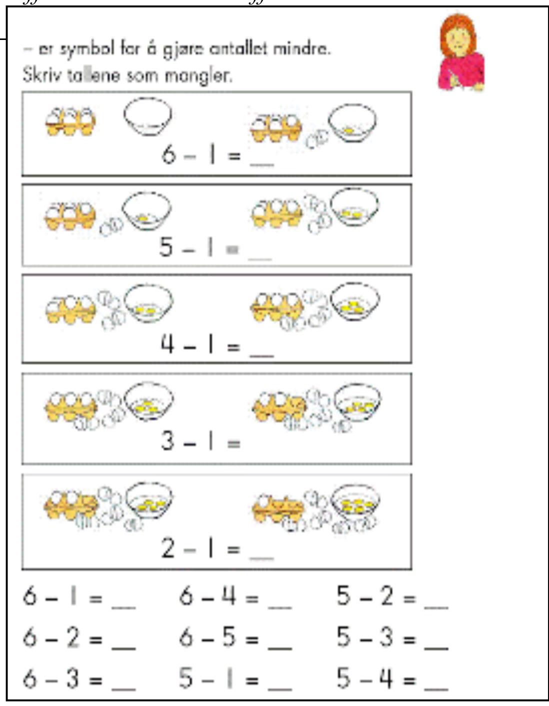


Figure 17. – is a symbol referring to decrease the number when you take away something.

Similar progresses are designed to work with other concepts necessary for understanding. In teaching we have often put impact on number concepts, and number understanding. We have been less concerned of concepts of number operations, like subtraction and multiplication; and number relations like number similarity. A learning progress, akin the one for subtraction, is designed to teach concepts about number similarities before the symbol = is introduced. More famous writers than me have described how children understand = as if the meaning was “here comes the answer” (Ginsburg 1996). If we examine math books we can find the reason. The children reflect exactly what they are taught.

Math books regularly have too many pages of exercise. It seems like we need to have available tasks which the children can work with in peace and quiet, with as little movement and noise as possible. This statement does not mean that I underestimate the importance of exercise. The children need to automate number combinations, like “all’ which makes seven” and “all’ which makes ten”. Only it is not sure that the best way to learn it is to write down calculations. Games can be more efficient when playing them includes verbal expressions. It is easy to make lotto games or memory games with the possible addition and subtraction combinations 1-20. Let the children play, but let the rule be: *Nobody gets a card without telling aloud what it says*. There are also available numberless items of computer programs. It is wise, also in work with these, to follow the rule of verbalizing, remembering Luria’s statement: “*with the aid of speech...”*

Analytic Coding.

Math books also should include tasks for analytic coding. Examples of this follows below.

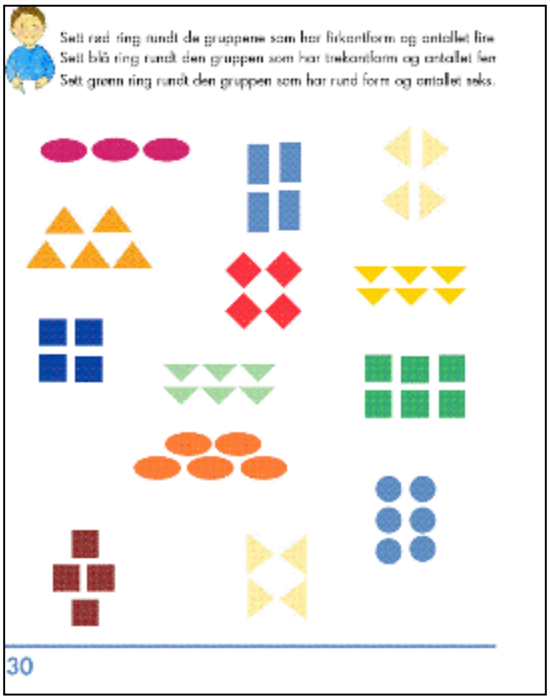
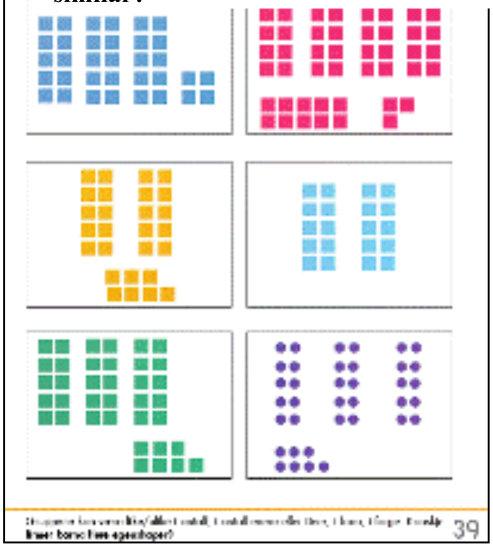


Figure 18. Put a ring around groups which have figures with a four-sided shape and the number four etc.

Figure 19. Look at groups placed beside each other. In what respects are they similar? In what respects are they not similar?



This may seem difficult for seven-year-old children. When they have learnt the prerequisites, reality is the opposite. To verbalize such similarities and differences contributes to direct attention towards important features and facilitates verbal consciousness or metacognition.

These examples are too few to give a broad picture of how it is possible to approach math learning taking into consideration theories about learning in general. So far, there is evidence that children learn to understand, and to be independent and autonomous in their learning and available to transfer their knowledge into new situations. Though, still this is a field in need of research.

Jaques Maritain, the French Canadian philosopher, considered education to be a dynamic enterprise. The learner is not passive clay, and the art of teaching is not like sculpturing, he stated, and continued:

> *This inner vital principle the teacher must respect above all; his (her) art consists in imitating the ways of the intellectual nature in its own operations. Thus the teacher has to offer to the mind ... examples from experience ...from which he (the pupil) will go on to discover broader horizons. The teacher has further to comfort the mind of the pupil by putting before his eyes the logical connections between ideas which the analytical or deductive power of the pupil's mind is perhaps not strong enough to establish by itself.*
>
> Maritain, 1979

My experience is that some professionals hardly believe; when they hear the story of Tina's learning progress. Maybe the successful part of it is owing to contexts that established a framework in which the intellectual nature in its own operations could come to work. Maybe the theoretical considerations, whether they come from Luria, Vygotsky, Nyborg, Feuerstein, or Das, have something to offer to the same extent to which they contribute to uncover and explain these intellectual nature's own operations. Finally, what we do to create exciting contexts for learning is successful as far as we manage to give the best conditions for the intellectual nature to work. To maintain being means to an end, the intellectual nature must be understood in terms of all human processes contributing to learning; with motivational and emotional aspects in addition to those this article has primarily had in focus.

Quoting Maritain, I think I have already answered the question posed in the title of this article. There are general learning processes important also for math learning. Rejecting so, would be to disregard those who through their research contributed to our understanding of learning; those on whose shoulders we stand in our work. It is wise to be aware of that, while we, instead of resting on their laurels, continue their work to the benefit of children's learning.

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